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Magneto-optical metamaterials with extraordinarily strong magneto-optical effect

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In optical frequencies, natural materials exhibit very weak magneto-optical effect. This weak effect can be easily recognized in the dielectric tensor where the ratio between the imaginary off-diagonal and diagonal components is extremely small. Here, we show that man-made metamaterials can greatly enhance this ratio, leading to an extraordinarily enhanced magneto-optical effect. © 2016 AIP Publishing LLC. [<http://dx.doi.org/10.1063/1.4945051>]

Nonreciprocal optics has important applications in optical isolators and circulators.¹⁻⁴ Recently, it is also used to explore intriguing topological properties of light such as the optical analog of quantum Hall effect.⁵ These devices and optical phenomena rely on breaking the time-reversal symmetry to realize nonreciprocal responses. Despite emerging methods such as dynamic modulation⁶ and optical nonlinearity,⁷ the magneto-optical effect remains the most effective way to break the time-reversal symmetry.^{8,9} Unfortunately, the magneto-optical effect is extremely weak in optical frequencies due to the intrinsic limit of natural materials.¹⁰ In this letter, we show that magneto-optical metamaterials that combine regular magneto-optical materials and metals could exhibit greatly enhanced magneto-optical effect. In contrast to the common way to enhance the magneto-optical effect through optical resonance,¹¹⁻¹³ the enhancement is realized by directly engineering the effective dielectric tensor^{14,15} of the metamaterials. These magneto-optical metamaterials could be useful in a variety of nonreciprocal photonic devices.

We start by discussing the strength of the magneto-optical effect by considering the polarizability tensor of a material

$$\vec{\chi} = \begin{pmatrix} \chi_d & 0 & 0 \\ 0 & \chi_d & -i\chi_o \\ 0 & i\chi_o & \chi_d \end{pmatrix}. \quad (1)$$

The imaginary off-diagonal component is responsible for the magneto-optical effect. It breaks the time-reversal symmetry and creates nonreciprocal responses. χ_o is zero in regular materials while it is non-zero in magneto-optical materials. Unfortunately, all natural magneto-optical materials have $\chi_o \ll \chi_d$ in the optical frequencies. Consequently, the optical response of the material is still largely controlled by the diagonal polarizability χ_d , which tends to overwhelm the magneto-optical response. We can use the polarizability ratio,

$$V = \frac{\chi_o}{\chi_d}, \quad (2)$$

to characterize the strength of the magneto-optical effect. Natural materials have very small $V < 1\%$ even under strong a magnetic field. For example, Bismuth iron garnet^{16,17} has a maximum $V \approx 0.01$ with $\chi_d = 5.25$ and $\chi_o = 0.06$.

The strength of the magneto-optical effect and the ratio V can be directly measured in a polar Kerr experiment. In such an experiment, a linearly polarized light is normally incident upon a magneto-optical material. Specifically, Fig. 1(a) shows an incident light with its electric field polarized along the y -axis $\vec{E} = E_0\hat{y}$. The reflected light is generally elliptically polarized. The complex Kerr angle measures the rotation of the polarization. It is defined as $\phi_K = E_z^r/E_y^r$, with $E_{z(y)}^r$ being the electric field of the reflected light in $z(y)$ direction. The real part of ϕ_K is the Kerr rotation and the imaginary part is the Kerr ellipticity.^{18,19} When $\chi_o \rightarrow 0$, the Kerr angle can be derived as

$$\phi_K \sim iV. \quad (3)$$

It is a direct measure of the strength of the magneto-optical effect. The small ellipticity observed in most natural materials is directly caused by the small V ratio. On the other hand, strong magneto-optical effect would lead to a large V and

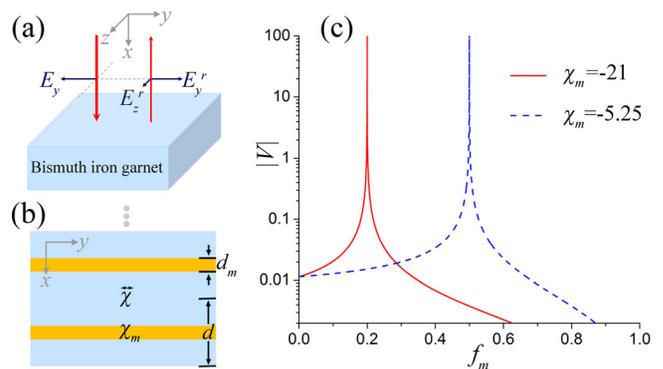


FIG. 1. (a) Schematic of the polar Kerr effect. (b) Schematic of the metamaterial structure. It consists of normal magneto-optical material such as Bismuth iron garnet (blue layer) and metal (yellow layer). χ_m and $\vec{\chi}$ are polarizabilities of the metal and magneto-optical material, respectively. (c) The calculated $|V|$ as a function of the filling ratio f_m when $\chi_d = 5.25$ and $\chi_o = 0.06$.

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Kerr angle. When $V \rightarrow \infty$, the reflected light is linearly polarized along the z -axis. It rotates by 90-degree compared to the incident light's polarization. Such extreme case has only been observed in materials under extreme conditions.²⁰

Next, we show that magneto-optical metamaterials can drastically enhance the magneto-optical effect. The structure of the metamaterial is shown in Fig. 1(b). It is a layered structure that combines a normal magneto-optical material and a metallic material. The thickness of each layer is less than 10 nm so that the metamaterial can be treated as a uniform medium for optical waves. It is periodically arranged with a period of d . The filling ratio of the metal is $f_m = d_m/d$ with d_m being the thickness of the metal. The filling ratio of the magneto-optical material is $f_{mo} = 1 - f_m$.

The dielectric response of the metamaterial can be tuned by changing the filling ratio. In particular, the V ratio of this composite magneto-optical material can be greatly enhanced. Next, we first use the effective medium theory^{21–26} to derive the effective polarizability tensor of this metamaterial. For this purpose, we apply the boundary conditions at the interface between different materials. For the direction parallel to the interface, the electric field $\vec{E}_{\parallel} = (E_y, E_z)$ is continuous. Thus, the averaged polarization in a unit cell can be calculated as $\vec{P}_{\parallel} = (f_m \chi_m + f_{mo} \vec{\chi}) \vec{E}_{\parallel}$. The corresponding average displacement field is

$$\vec{D}_{\parallel} = (1 + f_m \chi_m + f_{mo} \vec{\chi}) \vec{E}_{\parallel}. \quad (4)$$

For the direction normal to the interface, the displacement field D_x is continuous. The averaged electric field in a unit cell can be written as

$$E_x = \left(\frac{f_m}{1 + \chi_m} + \frac{f_{mo}}{1 + \chi_d} \right) D_x. \quad (5)$$

Combining Eqs. (4) and (5), we obtain the effective polarizability

$$\vec{\chi}_{eff} = \begin{pmatrix} \frac{(\chi_d + 1)(\chi_m + 1)}{f_{mo}\chi_m + f_m\chi_d + 1} - 1 & 0 & 0 \\ 0 & f_{mo}\chi_d + f_m\chi_m & -if_{mo}\chi_o \\ 0 & if_{mo}\chi_o & f_{mo}\chi_d + f_m\chi_m \end{pmatrix}. \quad (6)$$

The ratio between the diagonal and off-diagonal polarizability is

$$V = \frac{f_{mo}\chi_o}{f_{mo}\chi_d + f_m\chi_m}, \quad (7)$$

which can be tuned by changing the filling ratio f_m .

Considering that the metal has a negative polarizability $\chi_m < 0$, we could choose a filling ratio $f_m = \chi_d / (\chi_d - \chi_m)$ such that $f_{mo}\chi_d + f_m\chi_m \approx 0$. In this case, $V \rightarrow \infty$ and the optical response is dominated by the magneto-optical effect. As a specific example, we use Bismuth iron garnet together with a metallic layer. Fig. 1(c) shows the polarizability ratio $|V|$ as a function of the metal filling ratio f_m . For a metal with a polarizability $\chi_m = -21$, $|V| \rightarrow \infty$ when the filling ratio $f_m = 0.2$. For different metals with different polarizability,

we could also reach an infinitely large V by adjusting the filling ratio as long as the polarizability is negative. For instance, the dashed line in Fig. 1(c) shows the case with a polarizability of $\chi_m = -5.25$. V is drastically enhanced around the filling ratio of $f_m = 0.5$. We note that the specific layer configuration is not critical. The same enhancement effect can be observed as long as the filling ratio satisfies $f_{mo}\chi_d + f_m\chi_m \approx 0$.

In order to validate the effective medium theory, we use numerical simulation to study the polar Kerr reflection. The simulation solves the full-wave Maxwell's equations using a finite element method in the frequency domain. Fig. 2(a) shows the setup of the simulation. A thin metallic film is embedded in a thin layer of magneto-optical material with a total thickness of $d = 10$ nm. An incident light is polarized along y -axis. The wavelength is $\lambda = 750$ nm. According to the effective medium theory, the polarization in this thin metamaterial can be written as

$$\vec{P} = \chi_{eff} \vec{E} = \alpha \begin{pmatrix} 0 \\ 1 \\ iV \end{pmatrix}, \quad (8)$$

where $\alpha = (f_{mo}\chi_d + f_m\chi_m)E_0$. The induced polarization in the material determines the polarization of the reflected light. When $V = 0$, the reflected light is linearly polarized along the

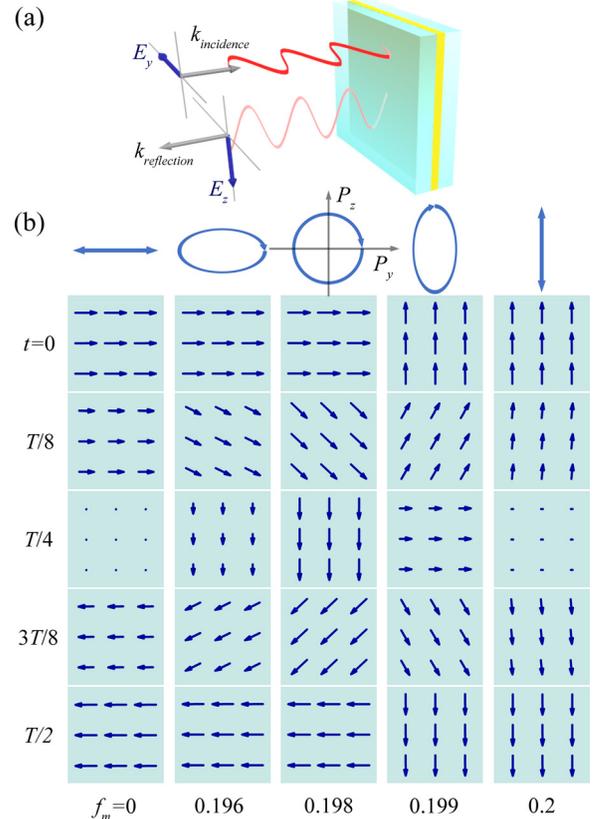


FIG. 2. (a) Schematic of the polar Kerr simulation. The structure consists of a metal layer embedded in a magneto-optical material. The thin film extends to infinity in both y - and z -axes. The polarizability of the magneto-optical material are $\chi_d = 5.25$ and $\chi_o = 0.06$. The metal's polarizability is $\chi_m = -21$. (b) The polarization state of the reflected light for different f_m . Lower panels show the time-evolution of the direction of the electric field for the reflected light.

same direction as the incident light. When $V \ll 1$ such as in natural magneto-optical materials, the reflected light is slightly elliptically polarized but with a very small z -component. When $V \rightarrow \infty$, the polarization of the reflected light is rotated by 90° and is linearly polarized along the z direction.

We confirm the above prediction in our numerical simulations. Specifically, we measure the electric field of the reflected light. The time evolution of the electric field over a half period of time $T/2$ shows the polarization state of the reflected light. We start with a film of magneto-optical material without any metal, i.e., $f_m = 0$ as shown in the first column of Fig. 2(b). The reflected light is polarized along the same direction as that of the incident light because of an extremely small $V \approx 0.01$. Next, a thin metal layer is embedded in the magneto-optical material. As we gradually increase the filling ratio of the metal f_m , the effective V also increases. Consequently, the reflected light becomes more elliptically polarized as shown in Fig. 2(b). When the filling ratio reaches a critical value of $f_m = 0.2$, a 90° of polar Kerr rotation is observed! This drastically enhanced magneto-optical effect is consistent with our effective medium theory.

In the above analysis, we have used infinitely large thin film to derive the effective polarizability. In fact, the conclusion applies equally to finite structures, which could be more relevant in practical applications.^{13,27,28} Figure 3(a) shows a nano-disk made from the magneto-optical metamaterials. We use full-wave simulation to study its scattering properties and show that the optical response also exhibits drastically enhanced magneto-optical effect.

In the simulation, we consider a nano-disk with a radius of 25 nm as shown in Fig. 3(a). A plane wave polarized along the y -axis is incident upon the nano-disk and the scattering field is calculated. We first study a disk made entirely of a magneto-optical material (i.e., $f_m = 0$). The far-field distribution (Fig. 3(b)) of the scattering field clearly shows a dipole radiation pattern. This radiation pattern implies that the induced dipole moment in the nano-disk is along the y -axis, i.e., the same direction as that of the incident light's polarization. In contrast, when we embed a metal layer in the magneto-optical material with a filling ratio of $f_m = 0.2$, the

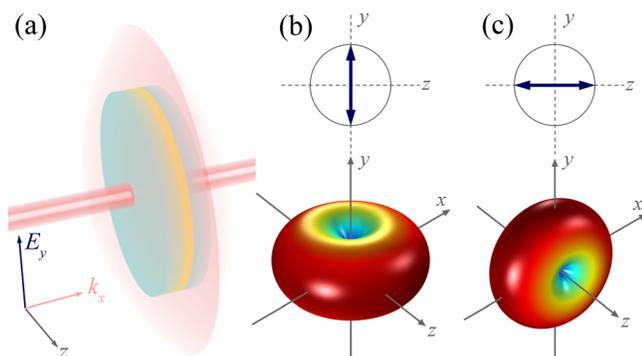


FIG. 3. (a) Schematic of the simulation for calculating the scattering field. The nano-disk consists of a metal layer embedded in a magneto-optical material. The wavelength of the incident light is 750 nm. (b) Induced polarization (upper panel) and the far-field distribution of the scattered field (lower panel) when there is no metal, i.e., $f_m = 0$. (c) Same as (b) with $f_m = 0.2$ and $\chi_m = -20.865$.

far-field distribution of the scattering field shows a completely different dipole radiation pattern (Fig. 3(c)). This rotated pattern implies that the induced polarization in the nano-disk is along the z -axis! The polarizability of the nano-disk is dominated by the magneto-optical response of the magneto-optical metamaterial.

Finally, we discuss the limitation of the magneto-optical metamaterials. First, the optical loss in metallic materials will reduce the enhancement factor. However, a significant enhancement can still be obtained with real metals. For example, we consider the same wavelength of 750 nm and use Bismuth iron garnet together with copper. The layered metamaterial can enhance the Kerr angle $|\phi_K|$ by 30 times with a metal filling ratio of $f_m = 0.208$. The Drude model $\epsilon_m = 1 - \frac{\omega_p^2}{\omega^2 + i\omega\gamma}$ is used to model copper with a plasma frequency of $\omega_p = 1.1 \times 10^{16}$ rad/s and an electron collision frequency of $\gamma = 0.9 \times 10^{14}$ rad/s.²⁹ Another limitation includes the finite operation bandwidth due to the strong dispersion of the metallic structures. This bandwidth can be broadened by using another dispersive material that can offset the change of dielectric constants. Finally, it is noted that the Faraday rotation can also be enhanced in magneto-optical metamaterials. But the enhancement factor is generally lower than that of the Kerr rotation angle.

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